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The Modelling of Panel Radiator Dynamic Behaviour

Modelování dynamického chování deskového otopného tělesa

This paper deals with analysis and modelling of thermal dynamic processes of a panel radiator. The analysis is based on observing the temperature field on the frontal surface of the radiator by a thermal infrared camera. Two different simulation models in Matlab Simulink are presented here. Both are made for the specific panel radiator (a classic type with one panel, 0.5 m height and 1 m long with a single side connection from the top to the bottom). The dynamic behaviour is usually determined by (expensive) measurements, so the main goal is to create such a model that will be simply able to simulate the dynamic behaviour as real as possible for certain similar types of radiators, with minimum input values.
Keywords: *Heating, dynamics of radiators, modelling, Matlab, control of heat power*

Příspěvek se zabývá analýzou a modelováním tepelné dynamiky deskových otopných těles. Úvodní experimentální analýza je založena na sledování teplotních polí na přední teplosměnné ploše otopného tělesa termovizní kamerou. Jsou zde popsány dva rozdílné simulační modely v softwaru Matlab. Oba jsou vytvořeny pro deskové otopné těleso „klasik" s jednou deskou (typ 10) o rozměrech 500 × 1000 mm (výška × délka) a s připojením k otopné soustavě jednostranně shora dolů. Dynamika je v současnosti spolehlivě zjišťována téměř výhradně nákladným (termovizním) měřením, a proto je základním cílem vytvoření takového modelu, který by byl jednoduše schopen generovat údaje o tepelné dynamice otopných těles co nejvěrněji a s dosažením minimálního počtu vstupních dat.
Klíčová slova: *Vytápění, Dynamika otopných těles, Matematické modelování, Matlab, Regulace tepelného výkonu*

INTRODUCTION

All the individual radiators are important accumulation elements in heating systems and, therefore, they could be imagined as resistors. The dynamics of the radiators can be described with their thermal inertia. The thermal dynamics (and inertia) were measured with an infrared camera (Flir ThermaCam type T460). The selected radiator was monitored under simplified laboratory conditions in a so-called open space (according to DIN 4704).

There are two modelling approaches described here, the processes of both are quite different. The first model is based on physical laws, where the heat output is determined only on the water side by the calorimetric equation. Then the heat output is shared through the wall of the radiator into the ambient air. The second model is the first order stochastic discrete black-box model. It identifies the dynamic parameters only on the basis of the real measured data without any prior information about the physical dependencies. The result of this approach is a discrete dynamic model, which is described by a differential equation.

Compared to the previously presented results [1], [2], there is a fundamental difference. The discrete black-box model is able to generate a complete dynamic picture for the transferred heat to the room at different temperature levels both within the heat-up and cool-down stages. The entire model is based on the unique record made by the thermal camera. The static properties for the different mass flow rates and temperature parameters for this model also were measured. Such dynamic models help to increase the efficiency of the control processes and controller designs for specific applications and different boundary conditions.

PHYSICAL MODEL

Experiment

A basic type of a panel radiator was selected for measuring: a single panel (type 10) with dimensions of 500 × 1000 mm. The radiator was connected in a nominal way – a single side from the top to the bottom. The nominal values of the temperature and mass flow rate were determined and set at the beginning. The thermal camera took an image once every five seconds – thus, a series of thermography images arose. The dependence of the radiator´s surface temperature against the time was obtained. The mean surface temperature of the radiator t_p and its dependence against time may be then evaluated within the area of the frontal projection surface of the radiator. We can say, that the surface temperature is equal to the temperature of the water – this simplification allows the fact that the heat transfer on the side of the water is more intense than the heat transfer on the side of the air. The thermal conductivity coefficient of the radiator´s body material is high and the thickness of the radiator´s wall is relatively small, at the same time. Therefore, the water temperature drop caused by the heat transfer on the water side and the heat conduction in the material can be neglected. The mean temperature of the water t_{wm} is then approximately equal to the mean surface temperature of the radiator t_p on the air side.

The dynamic processes were captured from the initial changes of the mean surface temperature (i.e., without any dead time) until the new steady state. Then, using the procedure described in [3], the characteristic curves can be converted from the dependence of the mean surface temperature against the time to the dependence of the relative heat output against the time. The resulting transient curves from the experiments, used to create the model, were presented, e.g., in [4].

Procedure

The methodology of the approximation of the measured processes was based on the procedure known as the approximation by Strejc ([5], [6]). It can be only used if the system response is not oscillating. The real function is approximated by a second order proportional system with two different time constants (when $\tau_u < 0.104$) or by an n-th order system with two equal time constants (if $\tau_u > 0.104$). The choice of the system depends on the value of the parameter τ_u. This parameter is typically less

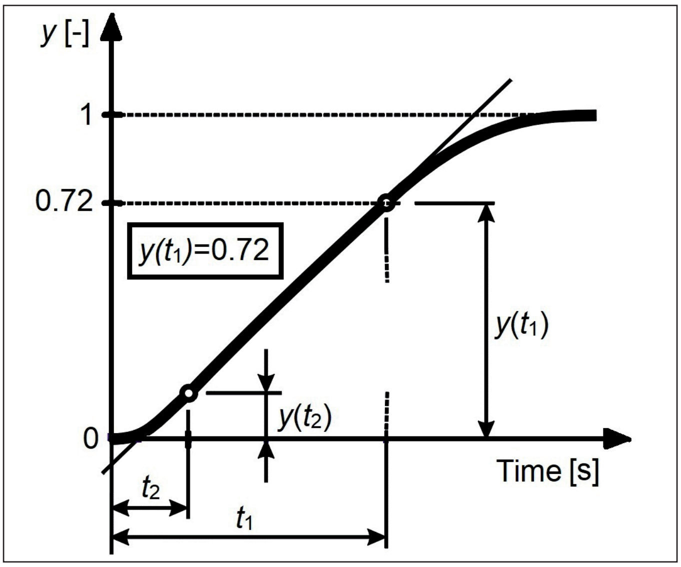

Fig. 1. The principle of determination of the time constants according to Strejc´s method

than 0.104 in the field of the radiator's use and, therefore, only the case of the 2nd order approximation will be described.

$$\tau_u = \frac{T_u}{T_n} \tag{1}$$

Where τ_u is the dimensionless parameter for the approximation selection; T_u is the process delay [s]; T_n is the process reaction rate [s]. If $\tau_u < 0.104$, then the final shape of the transmission function is according to Equation (2), where $\tau_{0,1}$ and $\tau_{0,2}$ are the individual time constants.

$$G_{(s)} = \frac{K}{\left(\tau_{0,1}s+1\right)\cdot\left(\tau_{0,2}s+1\right)} \tag{2}$$

The gain K is given by the ratio of the newly stabilised value of the output variable Δy to the steady value of the input variable Δx. The approximation theory is based on Equation (3).

$$y(t_1) = 0.72 \cdot y_{(\infty)} \tag{3}$$

Time t_1 is subtracted from the respective transition curve for the value $0.72.y$ and the sum of the time constants $\tau_{0,1}$ and $\tau_{0,2}$ is calculated according to Equation (4):

$$\tau_{0,1} + \tau_{0,2} = \frac{t_1}{1.2564} \tag{4}$$

After that, it is necessary determine time t_2 according to Equation (5) and the value of $y(t_2)$ is subtracted from the corresponding transient characteristic. An example is shown in Figure 1.

$$t_2 = 0.3574 \cdot \left(\tau_{0,1} + \tau_{0,2}\right) \tag{5}$$

Depending on the tabulated values (e.g., [5]), the ratio of the time constants τ_2 is determined. The constants $\tau_{0,1}$ and $\tau_{0,2}$ are then simply calculated from $\tau_2 = \tau_{0,2}/\tau_{0,1}$. It is easy to set time t_2 from Equations (4) and (5) at the same time.

Model progress

The change in the inlet water temperature flowing into the radiator was considered as the input to the model (as a step change). At the beginning, the radiator is temperature-balanced with its environment and then water of a nominal temperature (according to EN 442, it is 75 ° C) is introduced into it. The output of the model is the desired dependence of the mean surface temperature of the radiator against time, which can be converted into the dependence of the heat output against time by the procedure given in [3].

The entire approximation process is included in the model, and the determined values of the gain and time constants are then inserted into the transmission function. At the beginning of the development process,

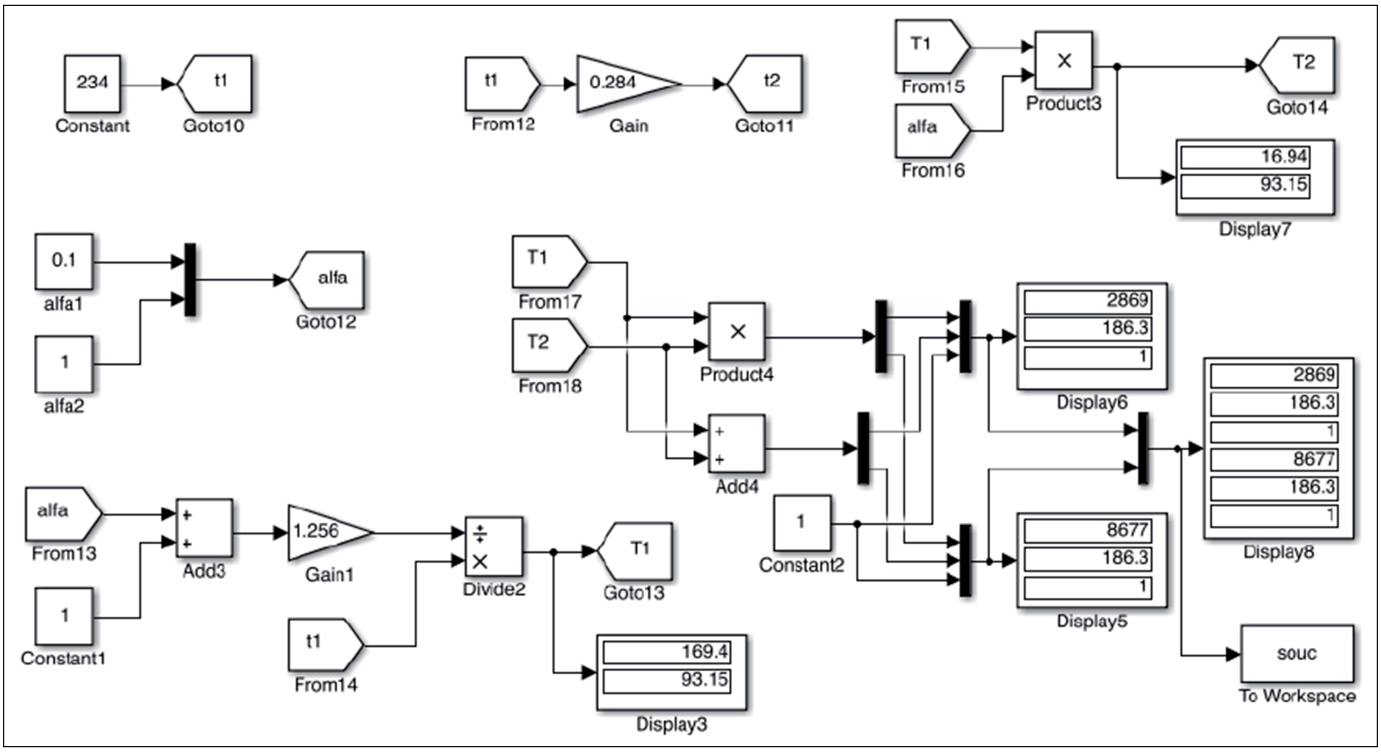

Fig. 2. The scheme of the approximation model for the 2nd order function

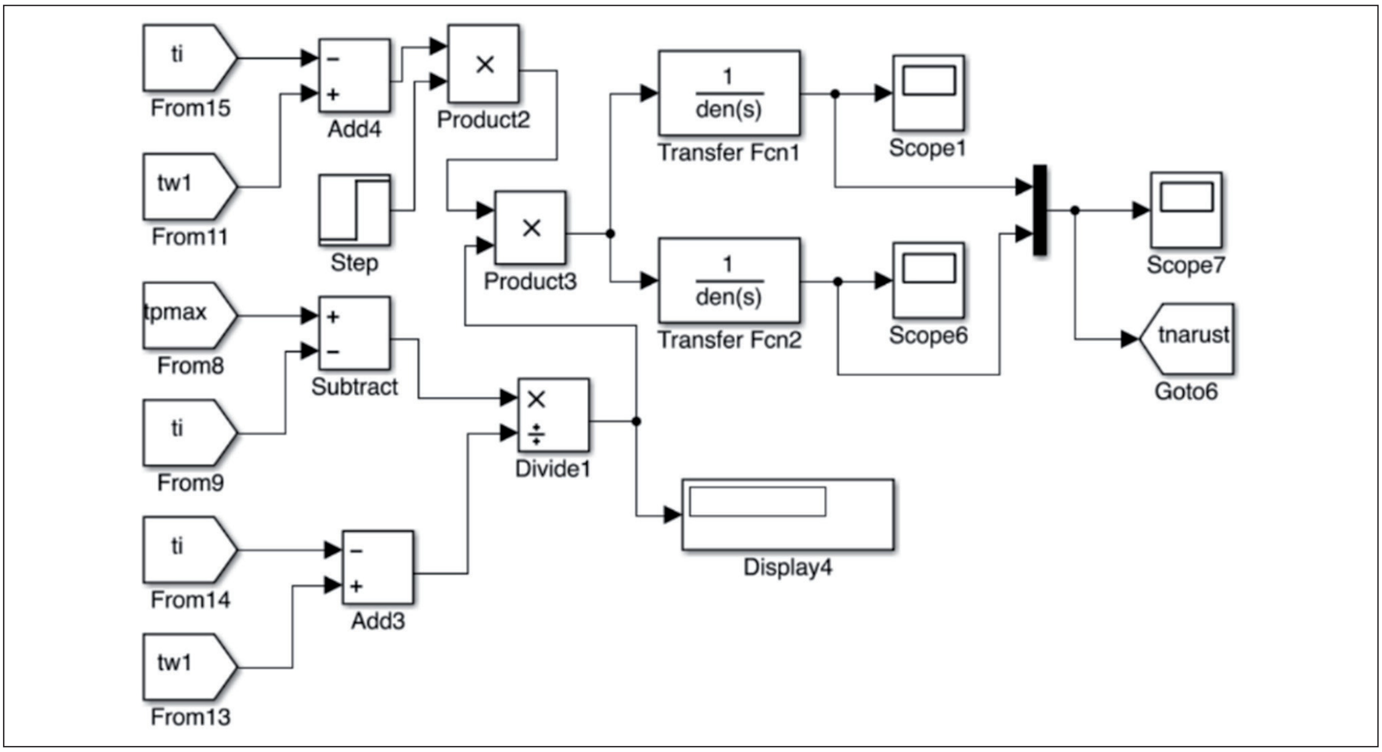

Fig. 3. The scheme of the model with the transmission functions

it was necessary to divide the model into two independent programmes that had to be run in the right order. The first of these is the programme for determining the function approximation (see Figure 2). Then, the simulation programme for mean surface temperature started, based on the approximated outputs (see Figure 3). A two-channel approach to the approximation calculation (it is allowed by Strejc´s method) has been introduced. In the first channel it is considered with the two time constants as mentioned above (because $\tau_u = 0.1$ and it is referred to block "alpha" in the model in Figure 2). The second channel of the calculation then considers that $\tau_u = 1$. This simplifying assumption leads to the calculation of the transmission functions with only one time constant. Such a solution leads to a more flexible approximation based on the input data and it is not necessary to work with one fixed transmission function only, but with the so-called flexible transmission.

It was a natural development to join up two consecutive programmes into one set, which would be able to connect the input values of the necessary variables and coefficients. This was achieved by replacing the blocks of the transmission functions with the integrator blocks. The integrator was introduced as a transmission block with a value of $G_{(s)} = 1/s$. The appropriate combination of these blocks and the constants can provide the same functions as the transmission ones.

Using the flexible transmission model for the heating-up of the radiators, the final model of entire dynamic process of the radiator was built. It was also necessary to build a cool-down phase model for this simulation. However, this is a simpler process than the heat-up in the mathematical description. Since the process delay is almost undetectable, the resulting transition curve may advantageously be referred to as a first order system. A significant source of the cool-down model uncertainty is the estimate of the heat transfer coefficients both on the water and air sides. Another disadvantage is also the need to insert a dynamic parameter for each radiator which represents the value of the mean surface temperature in time that corresponds to a 72% change between the original and the new steady state. This fact makes this model very difficult to use in practice because this value is generally unknown.

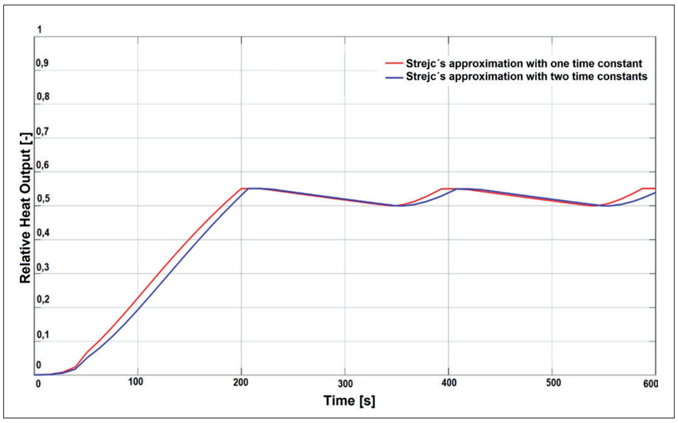

Fig. 4. The dynamic behaviour of the radiator – model. A heat output demand of 62.6 %; A range of proportionality of ± 1 K

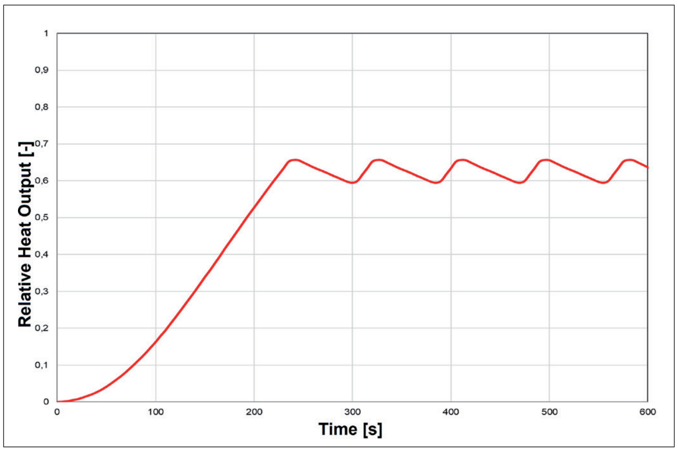

Fig. 5. The dynamic behaviour of the radiator – experiment. A heat output demand of 62.6 %; A range of proportionality of ± 1 K

It was necessary to link the processes of heating-up and cooling-down by a function, which is able to switch between them. We can use the "Relay" block in the Simulink environment. It disposes a two-position signal switching with hysteresis. In fact, this block can be imagined as a P-controller (i.e., a thermostatic liquid head).

Real systems always have some dead time which is introduced by the "Transport Delay" block. It has to be entered individually for each radiator within this model and, therefore, this is unfortunately impractical. After creation of the decision signal model, the complex dynamic behaviour of the radiator can be generated. An additional condition is that the temperature at the end of the individual phases is always used as the initial condition for the next phase. Finally, it was necessary to add a block of calculations for determining and setting the required range of proportionality within which the resulting control curves will move. This also gives the possibility to set the real range of proportionality of the actual P-controller that will be used. For lucidity, individual calculation blocks and other auxiliary calculation mechanisms (not illustrated in the figures above) were built into several subsystems that form a compact final model. The result is the mean surface temperature of the radiator and its heat output.

Results

Only small part of the graphical results from the experiments and the Matlab models are presented here. It is possible to simulate any heat output demand of the radiator within its power spectrum and any range of proportionality of the P-controller. The two curves (in Figure 4) are given by the two calculation channels with the different approach to the time constants. The dynamic behaviour compiled from the real measured data is shown in Figure 5. The conclusion of this physical model is listed below in the appropriate section.

DISCRETE BLACK-BOX MODEL

Experiment

The experimental measurements were not performed primarily for the purpose of determining the absolute values of the heat output, as usual, but, above all, for comparing the radiator's dynamic response and its behaviour in the different phases of the temperature spectrum.

Figure 6 presents a scheme of the measuring track with two independent heat sources. This arrangement is necessary to provide a (quasi) step change of the inlet water temperature. In addition, one of the sources is connected to the accumulation storage tank for increasing the temperature stability. There are also additional electrical heating cartridges in the storage tank. First of all, the mass flow rate was set, which corresponds to the nominal conditions specified by the radiators manufacturer. Both heat sources were connected to the by-pass at this moment. All the temperature changes were then performed at this constant mass flow. The dynamic response of the radiator on the mass flow rate changing was measured in a different measuring track configuration (with only one heat source). At the point, where the supply pipes from both sources meet, the desired temperature step change is ensured by means of a manually operated ball valve. The mass flow rates were measured by ultrasonic flow meters. The flow corrections were made according to the main flow meter, shared for both heat sources. Furthermore, the water temperatures at the inlet and outlet of the radiator were monitored.

So, the simple heating-up process between two steady states only was not observed (as for the physical model), but three stabilised temperature levels during the heat-up were provided. The inlet water temperature was changed to 50, then 60 and finally to 75 °C, at a constant mass flow rate.

It is not common in practise to change the inlet water temperature to the radiator in steps, but under laboratory conditions, it is a possible way to ensure the parameter that we are able to mathematically describe and evaluate. Then it is possible to observe and identify the dynamics of the radiators in the different phases of the temperature spectrum, and, what is the most important, it is possible to made models of their behaviour where the dead time is a necessary parameter representing the above-mentioned thermal inertia.

Figure 7 presents an evaluated record of the behaviour of a panel radiator from the experiment. It includes not only the mean surface temper-

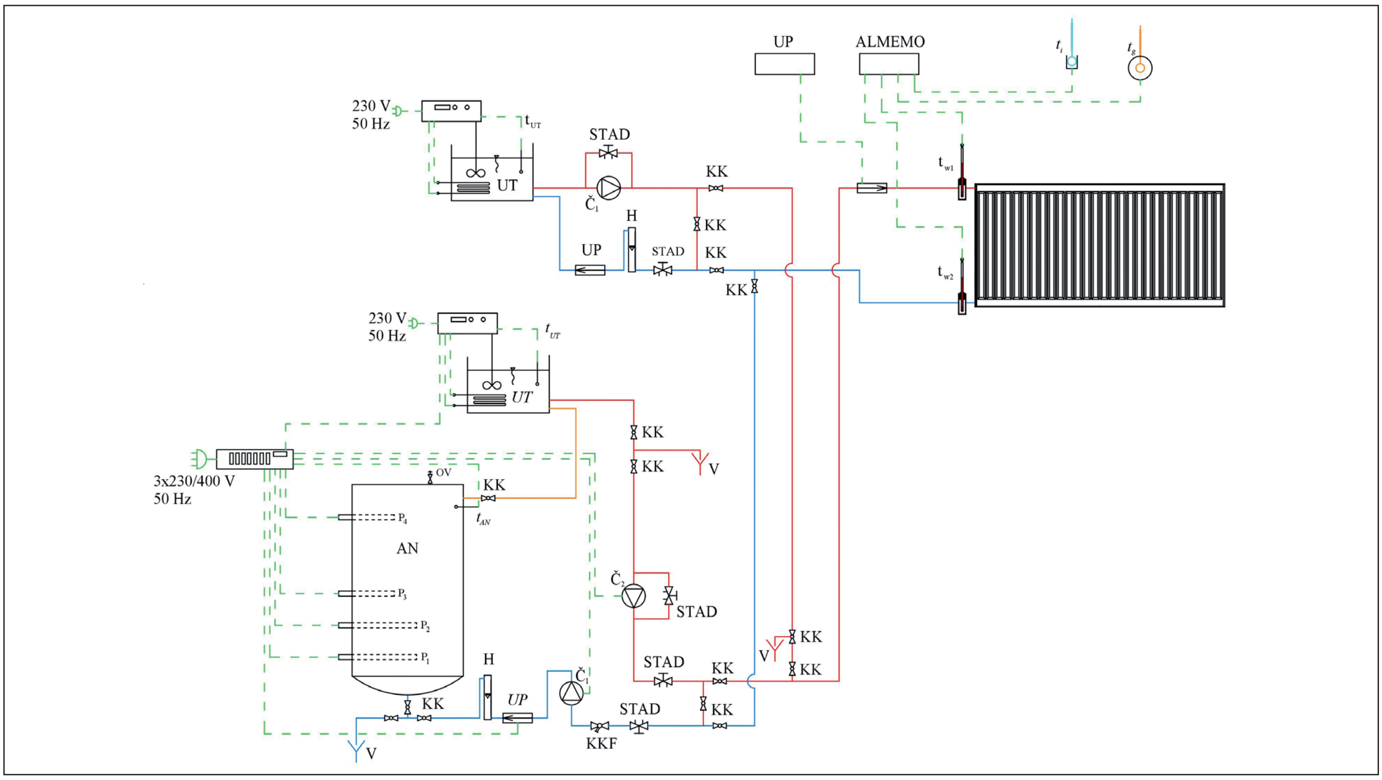

Fig. 6. The scheme of the experimental measuring track for the radiator with two independent heat sources

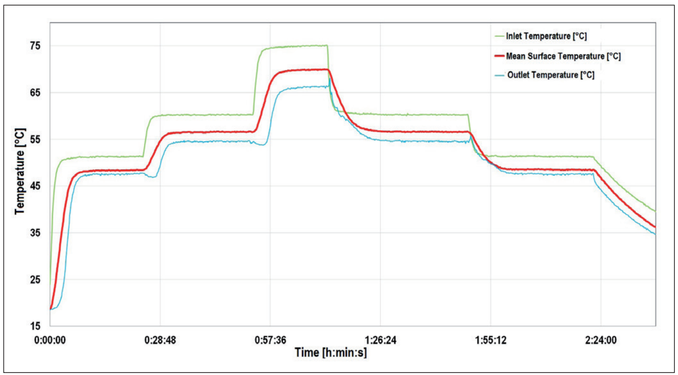

Fig. 7. The record of the operational dynamic behaviour of the radiator 10 – 500 × 1000 as a background for the model

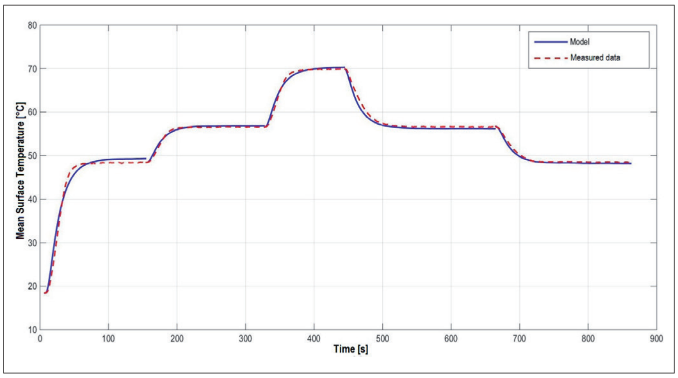

Fig. 8. The validation of the black-box discrete stochastic ARX model for the dynamic behaviour of the radiator

ature, but also the inlet and outlet water temperatures. It is clear that the changes in the inlet water temperature are almost steps. Unlike the outlet temperature, where due to inappropriately manipulation with the valve, we can observe a short-term negligible decrease. This drop at one point in the measurement does not have an effect on the mean surface temperature of the radiator.

Model progress

The basic idea is that the continuous system or its dynamic effects are approximated by a discrete model with a general transmission $G(z)$ with a suitable selected sampling time period. After careful analysis of the mathematical identification options, an ARX model approach (**A**uto**R**egressive with an e**X**ogenous variable) was chosen. AR models are generally able to describe random auto regression processes of any order. The discrete output values always depend on the actual input value and on the past output values that are weighted by the appropriate coefficients - hence the name of the model - autoregressive. The entire process of the mathematical identification using the ARX model is beyond the scope of this contribution and can be found in, e.g., [7]. If the transmission function is set as $G_F(z\text{-}1) = 1$, then the final derived equation of the discrete dynamic system for the model can be written in the following differential form:

$$t_p \cdot (\tau + 1) = a \cdot t_p(\tau) + b \cdot t_{w1}(\tau) \quad (6)$$

Where t_p is the mean surface temperature of the radiator against time τ [°C]; τ is the time [s]; a and b are the dimensionless coefficients of the differential equation; t_{w1} is the inlet water temperature [°C]. A sampling time period of 10 s was chosen.

Only the first-order polynomial was used for the simplest possible expression of the so-called Z-transformation and the subsequent script in Matlab. At the same time, the results of this model simulated the real measured dynamics of the radiator with satisfactory accuracy, see the following section.

It can be stated that the ARX identification method is based on the smallest square method. Basically, the point is to minimise the sum of the quadratic deviations of the estimated parameter vector (the real set of the mean surface temperature values $t_p(\tau)$) from the real measured values. A complete description of the so-called predictors – special vectors intended for the parameter estimation is described in [8].

Results

Figure 8 shows the comparison of the results obtained by the mathematical model and the real measured course. The maximum deviation from the measured values is up to 2 K of the mean surface temperature. As stated above - the model uses the simplest approach – a first-order function. In order to provide better accuracy of the mathematical model, it is possible to use an approximation of the second (or higher) order, where the deviation would be reduced below 1 K. However, those models require a much more complicated mathematical expression and a more complex controller design for their analytical solution.

Since a deviation up to 2 K is sufficient for the purposes of designing controllers of the radiator's heat output, the further specification of the mathematical model by a higher order approximation is no longer necessary.

CONCLUSION

Physical model

The physical model is only limited to a specific panel radiator type 10 – 1000 × 500 and to specific ambient conditions. To extend the usage of this model, it is necessary to further investigate the behaviour of other types of radiators under other conditions. It means further experiments. The approximation was carried out according to the method by Strejc. This means that it is necessary to enter the time when the 72% change in the transition function occurs. However, this value is not normally supplied by radiator manufacturers.

The displayed output from the model in Figure 4 shows a greater thermal inertia and lower relative heat outputs at the same time. The reasons for these deviations lie in the fact that a constant ambient temperature is introduced for the model. In this case, it was necessary to choose the mean surface temperature as the controlled variable. The range of proportionality, thus, relates to this value and not to the ambient air temperature. To solve it, it is necessary to construct a model of the entire space in which the radiator is located. Thus, it will be possible to simulate heat flows in such a space and to influence the varying ambient air temperature. The ambient air temperature then could be used as the feedback through the connected model of the P-controller. This will be the subject of the further development of this model.

Discrete black-box model

An interesting fact is the effect of the difference between the inlet water temperature and the mean water temperature in the radiator on its thermal inertia. It has been confirmed that at the higher value of this difference, the faster the processes take place. This fact is confirmed by the trend in all parts of the spectrum. This could be described by the time constant, which is the highest for the temperature change from 50 to 60 °C (3 min) and the lowest (2 min 45 s) for changing the temperature from the ambient temperature to 75 °C (not illustrated in Figure 8). Although the differences in the time constants are very small and sensitive to a proper evaluation, it is necessary to take this fact into account for the dynamic behaviour models. Of course, the specific thermal capacity of the radiator has a significant effect.

The rate of the thermal change also depends on the mass flow rate of the water. In order to construct the model, it was also necessary to map the behaviour of the radiator for the different flows. Thus, so-called static characteristics arise. The very practical knowledge was confirmed – the quality control, i.e., the change in the inlet water temperature is more effective than the regulation quantitative (varying mass flow). While the control of the heat source and the heating system is mostly qualitative, the local control of the heat output of the radiator is ensured by a quantitative change (P-controller) and, consequently, by a different water temperature drop inside the radiator.

It is evident that controlling the mass flow is not very effective for increasing the mean surface temperature, i.e., the heat output of the radiator. Manufacturers of thermostatic control valves can only partially compensate this effect by different valve characteristics. Generally, we should choose such parameters that ensure that a certain change of mass flow causes the same change in the heat output. The goal is linearity. Therefore, it is necessary to focus on the influence of the inlet water temperature. From the measured data, it is obvious that (for any mass flow) any increase in the inlet water temperature causes an almost linear increase in the heat output. From this point of view, it can be stated that the lowest inlet water temperature to the radiator is preferred, because the higher the inlet water temperature approaches the indoor air temperature, the more the dependence between the mass flow and the heat output is linear. Based on the above-mentioned analysis (in terms of the effective operation of the radiators and the circulation pumps), it is recommended to provide an inlet water temperature in the range from 50 to 65 °C and a temperature gradient in the radiators from 15 to 20 K. This is why it is very preferable to use condensing technologies, renewable heat sources or heat pumps. It is erroneous to assume that low-temperature heating systems only include a floor or wall heating. Heating systems with radiators, for today's building, envelope properties and can be designed as low-temperature systems without any problems with, e.g., the size of the radiators.

The black-box model can be applied to other radiators, but they have to be made with the same material (steel), with the same number of panels and with a length/height ratio of a radiator of no more than 3. For other radiators, the model needs to be verified by further experiments. This model can help with finding the appropriate value for the settings of the controllers suitable for the specific type of radiator.

In the long-term research, there are effort to create a universal model that provides an overview about the dynamics of different types of heating surfaces. However, because of their great diversity, it is clear that such a model is very difficult to make. The aim of this article is to make the readers acquainted with the possibilities of modelling a radiator´s behaviour.

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